Valuing Bonds

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Introduction

Bonds are contracts to repay borrowed money with interest at fixed intervals. For the issuer it is a way of raising capital from investors and maybe an alternative to sell additional shares of common stock.
Bond issuer could be a company, a municipality or a national government. We will focus on government bonds as investments and how to value them.

Calculating the present value of Bonds

Valuing a German long-term ”bund”

”Bundeslanleihen” pay interest (coupon) and principal in Euros. They mature after 10 or 30 years. For example, it is September 2010 and we think about buying €100 face value of a 4% bond maturing in July 2016.

Every year an interest payment of 4% = €4 will be paid, called the bond’s coupon. At maturity the principal of €100 will be paid back additionally.

Let us have a look at the cash flows:

Cash Flows (€)					
2011	2012	2013	2014	2015	2016
€4	€4	€4	€4	€4	€104

We assume that other German government bonds actually offer a return of 3.2%. Hence we discount at 3.2%:

$$PV = \frac{4}{1.032} + \frac{4}{1.032^2} + \frac{4}{1.032^3} + \frac{4}{1.032^4} + \frac{4}{1.032^5} + \frac{104}{1.032^6} = €104.31$$

We can say that our bond is worth €104.31 or 104.31% (of the face value).

We can use the annuity formula to value the coupon payments and add the present value of the principal payment as a shortcut:

$$PV = C_1 \left(\frac{1}{r} - \frac{1}{r\,(1+r)^t}\right) + \frac{C_2}{(1+r)^t}$$

$$PV = 4 \left(\frac{1}{0.032} - \frac{1}{0.032\,(1.032)^6}\right) + \frac{100}{1.032^6} = €104.31$$

If we buy the bond for €104.31 and hold it to maturity, we will earn a return of 3.2% over the six years. We say we get a **yield to maturity** of 3.2%.

Financial Newspapers like The Wall Street Journal publish daily prices at which bonds can be sold or bought. The stated ask yield corresponds the yield to maturity we could earn if we bought for the shown ”asked” price.

Valuing semi-annual Coupons and Bond Prices

U.S. government notes are bonds with a maturity of 10 years and semi-annual payments of interest.

Imagine a 4.5% note with a face value of $1,000 and an ask yield to maturity of 4.98%. It matures in 3 years from now. The yield over six month would be 4.98% / 2 = 2.49%. We have to discount the semi-annual cash flows of $22.50 with this ”semi-annual” yield.

$$PV = \frac{22{,}50}{1.0249} + \frac{22{,}50}{1.0249^2} + \frac{22{,}50}{1.0249^3} + \frac{22{,}50}{1.0249^4} + \frac{22{,}50}{1.0249^5} + \frac{1022{,}50}{1.0249^6} = \$986{,}69$$

Bond Prices vary with interest rates.

The interest rate *r* in the present value formula above represents the yield we could earn with similar investments at the actual market situation. If this interest rate falls, the calculated present value will increase, because our bond becomes more attractive compared with alternative investments.

The effect on the value of distant cash flows is higher than on short-term cash flows. Thus long-term bonds vary more by the effect of changing interest rates.

The weighted average time to each payment is called **duration**. The lower the coupon of a bond the longer the duration is (same maturity and face implied), because of the heavy weighted face value in the last year.

The **volatility** indicates how much the value of a bond will change if the yield varies by 1%. If there is a volatility of 3% and the interest rates increase by 0.5%, the price of the bond decreases by 1.5%. The higher the duration the higher is the volatility.

$$Volatilty\ (\%) = \frac{\text{duration}}{1 + \text{yield}}$$

Term structure and the expectations theory

Today's interest rate for a one-period loan is called one-period **spot rate**. There are different spot rates for different maturities, expressing the term structure of interest rates.
Imagine an upward-sloping term structure, this means the spot rates for long-termed bonds are higher than for the short-termed bonds. This is a usual situation.

In this case you will get an extra return for investing in a longer-termed bond. This additional return is named the **forward interest rate** and is calculated as follows:

$$f_2 = \frac{(1 + r_2)^2}{(1 + r_1)} - 1$$

Imagine a forward interest rate of 6 per cent for investing our money 2 years instead of 1 year. Our decision will depend on our expectation concerning the one-year spot rate ${}_1r_2$ at the end of year one. If we expect this rate to be higher than our forward interest rate (e.g. 7%) we will not want to hold a two-year bond. We will want to get another one-year bond at the end of the first year to gain the 7% interest rate.

Everybody with the same expectation will do so, and in a well-functioning market the price for the two-year bonds will fall until the forward interest rate will correspond with ${}_1r_2$. This is the **expectations theory** of the term structure.

Conclusions regarding the expectations theory:

- If investors expect short-term interests to rise, this will cause an upward-sloping term structure.
- If investors expect short-term interests to fall, this will cause a downward-sloping term structure.

The expectations theory is controversial, but it seems sure that expectations have an effect on the term structure.

The expectation theory does not consider risks. It is obvious that somebody who invests in long-term bonds with a higher risk wants these risks to be compensated by higher interest rates. This explains the higher interest rates of long-term bonds compared to those of short-term bonds.

Risks

The issuer goes bankrupt

If the issuer of a bond goes bankrupt, the invested money of the bondholders may be lost completely in the worst case. In general lending to a company is a higher risk than lending to a government, thus the interest rates of company bonds are higher.

Future level of interest rates

We learned that long-term bonds are more volatile than short-term bonds. So the risk of increasing interest rates can be reduced by investing in short-term bonds or by holding the bonds until maturity. On the other hand this reduces the possibility to benefit from decreasing interest rates.

Inflation

If we own a long-term bond we know what money we will get until maturity. But we do not know the purchasing power of the returned amounts at the time of maturity, which is determined by the rate of inflation.
The risk of increasing inflation can be reduced by investing in short-term bonds. The reason is that interest rates typically will be above the rate of inflation, so you will be able to reinvest your money at a higher interest rate than before, if inflation rises.
Another alternative is the investment in indexed bonds, these will be explained in the next paragraph.

Real and nominal rates of interest

For example a one-year bond returns a nominal cash flow of €105. Over the year the consumer price index shows an increase of 3%. The €105 at the end of the year will be able to pay goods as 105/1.03 = €101.94 could pay today.

The *nominal* payoff is €105, but the *real* payoff is only €101.94.

$$Real\ cash\ flow_t = \frac{\text{nominal cash flow}}{(1 + \text{inflation rate})^{\text{t}}}$$

In our example the nominal rate of return is 5%, but the real rate of return is 1.94%.

$$1.0194 = \frac{1.05}{1.03}$$

$$1 + r_{real} = \frac{1 + r_{nominal}}{1 + inflation\ rate}$$

We can also express this the other way around: if we expect a certain inflation rate we will want to get a higher nominal rate of interest. It is easy to adapt the formula above to get the wanted nominal rate:

$$1 + r_{nominal} = (1 + r_{real})(1 + inflation\ rate)$$

If we want 5% *real* interest in times of 3% inflation rate we have to demand a nominal interest rate of

(1 + 0.05)(1 + 0.03) -1 = 8.15%

We can eliminate the risk of an increasing inflation rate by buying indexed bonds. The real cash flows of inflation-indexed bonds are fixed, so the nominal cash flows depend on the changes of the inflation rate.