# How to Calculate Present Values

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## Present Value

### What is the Present Value

The Present Value is the value today of tomorrow’s cash flows. It is based on the fact that a Euro tomorrow is worth less than a Euro today. The problem is, that there is often more than one alternative to investing money. The main issue is to choose the best decision, to get the best results for the investment.

First we define the formula for the Present Value (PV):

$$PV = DF_1 \times C_1 = \frac{C_1}{1 + r_1}$$

$DF_1$ is the dicount factor, $C_1$ is the Cash flow and $r_1$ is the opportunity cost for a one year investment.

Example 1: You had the possibility to buy or to build an apartment by investing 700,000 € today and the chance to sell it in 1 year for 800,000 €.
On the capital market you will get a secure yield of 6 % p.a. The question is: Is this worth it?

PV = 800,000 € / 1+6% = 754,716.98 €

Net Present Value

-700,000 € + 754,716.98 € = 54,716.98 €
Because of the positive Net Present Value the investment is profitable and worthwhile.

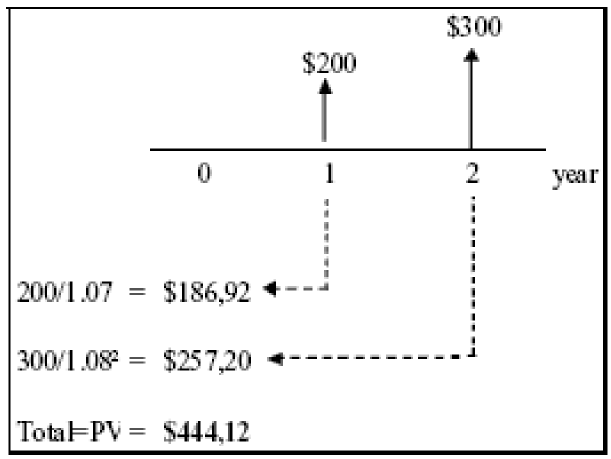
Example 2: You get a cash flow of € 2,000,00 for an investment over three years and the interest rate is 7 % p.a. How much is the PV?

PV = 2,000 € / $(1+7\%)^3$ = 1,632,60 €

## Valuing Cash Flows in Several Periods

The nice thing about Present Value is that they are all expressed in current Dollars, Euros or Kronor - so you can add them up.

Example 3: You are offered an investment that produces a cash flow of \$200.00 in year 1 and further a cash flow of \$300.00 in year 2. The interest rate rises from 7 % in year 1 up to 8 % in year 2.




The Present Value can add together.

$$PV = \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)} = \frac{200}{1.07} + \frac{300}{1.08^2} = \$444.12$$

We can continue to expand the formula for some more periods. This is called the *discounted cash flow* formula.
A shorthand way to write it is:

$$PV = \sum \frac{C_t}{(1+r_t)^t}$$

Example 4: You expected to receive a dividend of €100.00 in 5 years from a fairly stable investment.
1. The size of the cash flow is €100.00.
2. The risk seems to be very low. Similar investments to our dividend, paying companies require a 10 % return.
3. We have to wait 5 years.

Present Value = €100,00 / (1+ 10%)$^5$ = €62.09

In layman's terms, if you auctioned off the right to receive this €100.00 dividend in a public market, another investor would buy it for about €62.09.

## Calculating Present Values and Net Present Values

You want to calculate the Net Present Value of an investiment in real estate. You are able to purchase a property for €1 Million. You have to invest €120,000 p.a. during the first two years. After two years the building is complete and will be worth € 1,500,000.

All this yields a new set of cash-flow forecasts:

| **Period** | **t = 0** | **t = 1** | **t = 2** |
|---|---|---|---|
| Property | -1.000.000 | | |
| Invest | | -120.000 | -120.000 |
| Payoff | | | +1.500.000 |
| Total | $C_0$ = 1.000.000 | $C_1$ = -120.000 | $C_2$ = +1.380.000 |

If the interest rate is 7 %, then NPV is

$$
\begin{aligned}
NPV &= C_0 + C_1 / (1+ r) + C_2 / (1 + r)^2 \\
&= -1{,}000{,}000 - 120{,}000 / 1.07 + 1{,}380{,}000 / (1.07)^2 \\
&= \textbf{€ 93,195.91}
\end{aligned}
$$

Since the Net Present Value is positive, you should still go ahead. The investment is profitable, but be careful if the forecast is risky or the opportunity cost of capital could increase. The NPV at r = 12 % is negative, the project should be rejected.

# Perpetuities and Annuities

## Perpetuities

Perpetuity is a stream of cash payment that continues forever. The constand payment starts at a fixed date.

Present Value of Perpetuity = Cash Flow / Return

Example 5: You can engage in an investment which guarantees you a cash flow of €1 billion each year. The interes rate is 10 %. What is the maximum sum you would like to offer?

PV = €1 billion / 0.10 = €10 billion.

## Annuities

An Annuity is an asset that pays a fixed sum each year for a specified number of years.

$$\text{Present value of annuity} = \frac{1}{r} - \frac{1}{r(1+r)^t}$$

Example 6: Suppose that you can pay for a brand new Volvo €5.000 at the end of each of the next five years and no cash down. The interest rate is 7 %. How much money does the car *really* cost?

PV of annuity = € 5.000 [1 / 0.07 – 1 / 0.07 $(1.07)^5$]
= € 5.000 x 4.100 = € 20,501

Example 7: Suppose the opposite. You get credit/ a loan from the bank of €25,000 to pay the car cash. The bank requires you to repay it in equal annual installments over the next 5 years. The interest rate of the bank is 14 %.

5 year annuity factor = [1 / 0.14 – 1 / 0.14 $(1.14)^5$] = 3,433
Annual payment = € 25,000 / 3,433 = 7,282.26

The rate is used to reduce the credit and is used to pay for the interest of the bank.

## Future Value of an Annuity

Sometimes it is necessary to calculate the future value of a level stream of payments, you have to calculate the Present Value by multiply by $(1+r)^t$.

$$Future\,value\,of\,annuity = \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \times (1+r)^t = \frac{(1+r)^t - 1}{r}\right]$$

Exaple 8: Suppose you plan to buy a boot in 5 years. You could save € 20,000 a year and earn a return of 8 % on these saving. How much will you have available to spend after five years?
First - how much your savings are worth today?

PV = € 20,000 [1 / 0.08 – 1 / (0.08 x $1.08^5$] = € 79,854

If you invested € 79,854 today, you just multiply by $1.08^5$

Value at the end of year 5 = €79,854 x $1.08^5$ = €117,332

## Shortcuts: Growing Perpetuities and Annuities

When you need to value a stream of cash flows that grows at a constant rate or you need to value an x-year stream of cash flows that grows with a rate g, you can use the following shortcuts:

| Year: | 1 | 2 | 3 | 4 | 5 | ... | Present Value |
|---|---|---|---|---|---|---|---|
| 1. Growing Three-year annuity | \$1 | \$1x(1+g) | \$1x$(1+g)^2$ | | | | $\frac{1}{r-g} - \frac{1}{(r-g)(1+r)^3}$ |
| 2. Growing Perpetuity A | \$1 | \$1x(1+g) | \$1x$(1+g)^2$ | \$1x$(1+g)^3$ | \$1x$(1+g)^4$ | ... | $\frac{1}{r-g}$ |
| 3. Growing Perpetuity B | | | | \$1x$(1+g)^3$ | \$1x$(1+g)^4$ | ... | $\frac{1}{(r-g)(1+r)^3}$ |

## Compound Interest and Present Values

There is an important distinction between compound interest and simple interest.

When money is invested at compound interest, each interest payment is reinvested to earn more interest in subsequent periods. The following Figure shows useful the progression of the balance.

| **Simple Interest** | | | **Compound Interest** | |
|---|---|---|---|---|
| Year | Starting Balance + Interest | Ending Balance | Starting Balance + Interest | Ending Balance |
| 1 | € 100 + 10 | € 110 | € 100 + 10 | € 110 |
| 2 | € 110 + 10 | € 120 | € 110 + 11 | € 121 |
| 3 | € 120 + 10 | € 130 | € 121 + 12,1 | € 133,1 |
| 10 | € 190 + 10 | € 200 | € 236 + 23,6 | € 259,6 |
| 100 | € 1.090 + 10 | € 1.100 | € 1.252.783 + 125.278 | € 1.278.061 |

Example 9: Suppose you invest €1 at a continuously compounded rate of 10 % (r = 0.1) for two years (t = 2). The final value of the investment is €121.

The value for the same investment but for ten years is €259,60.

The return at simple interest after two years would be about the same €120, but after ten years it is only worth €200, this is worth thinking about.