Valuing Long-Lived Assets

Oliver Trabalski, 2008-09-30

This chapter explains how you can calculate the present value of cash flow. Some very useful shortcut methods will be shown. These shortcuts provide a good opportunity for valuating cash flows. There are different types of cash flows. Some cash flows run for a specific period (annuities). Others have no specific end date (perpetuities). Furthermore, the valuation of growing cash flows will be shown and how to valuate them.

Cash flow - Short Introduction

First, we define the formula to use for an asset that produces a one year cash flow.

$$PV = DF_1 \times C_1 = \frac{C_1}{1 + r_1}$$

DF_1 is the discount factor for year one and r_1 is the opportunity cost for a one year investment.

In the formula, r_1 is the opportunity cost of a one year investment. Assume that a one year U.S. Treasury is 7%.
The present value is:

$$PV = \frac{C_1}{1 + r_1} = \frac{200}{1.07} = \$186{,}92$$

Use this primary formula to calculate cash flow in year two:

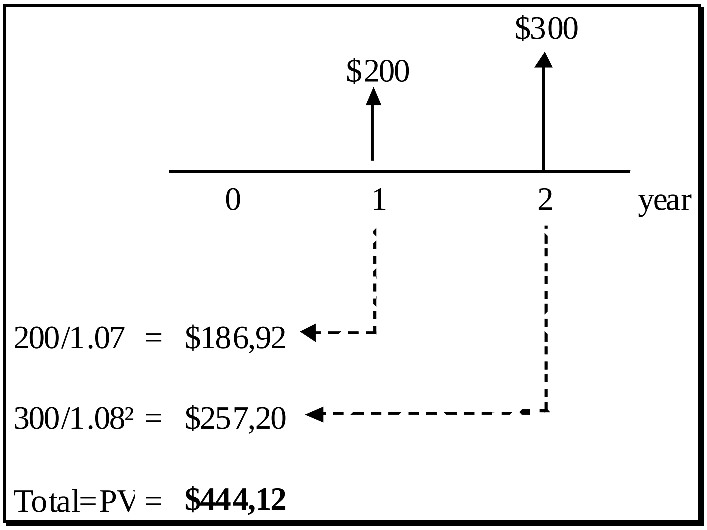
$$PV = DF_2 \times C_2 = \frac{C_2}{(1 + r_2)}$$

Suppose you get a cash flow of \$300 in year two (C_2=300) and the interest rate on a two year treasury is 8% per year. This means that a dollar you invested in two year notes will grow to 1.08^2=\$1.16 by the end of two years. The PV of year-2 cash flow equals:

$$PV = \frac{C_2}{(1 + r_2)^2} = \frac{300}{(1.08)^2} = \$257{,}20$$

Valuing cash flow in several periods

When you want to bundle present values through several periods you can add them very easily.



Present Value of an investment (2 years)

Now we can add present values to the total present value of an investment. The rule is:

$$PV = \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)} = \frac{200}{1.07} + \frac{300}{1.08^2} = \$444.12$$

Hence we can use the additive rule for extended stream of cash flows:

$$PV = \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)} + \frac{C_3}{(1+r_3)} + ...$$

This formula is known as the discounted cash flow (DCF). A shorthand way to write this formula is:

$$PV = \sum \frac{C_t}{(1+r_t)^t}$$

Calculation of Present Values and Net Present Values

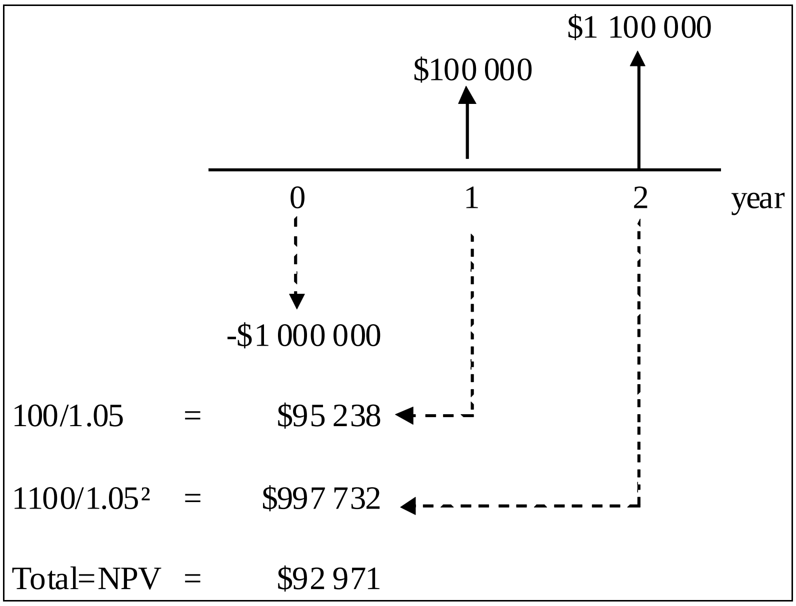
Now we want to calculate the net present value of a real estate investment. We will purchase a property for $1 million. In the first year we earn $100,000 and $150,000 in the second. At the end of the second year the board decides to sell the property for $950,000. The interest rate is 5%.

Period (t)	**0**	**1**	**2**
Purchase price	-1 000		
Income		100	150
Payoff			950
Total	C_0= -1 000	C_1= 100	C_2= 1 100

Example figures: thousands $

$$NPV = C_0 + \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)}$$

$$NPV = -1{,}000 + \frac{100}{1.05} + \frac{1{,}100}{(1.05)^2}$$



Calculation showing the net present value of the investment

In this case, the investment is profitable due to the positive NPV.

Valuation of Annuities and Perpetuities

Perpetuities

Perpetuity is a constant and never ending stream of payments. The constant payment starts at a fixed date.

$$Present\ value\ of\ perpetuity = \frac{C}{r}$$

Example: Present value perpetuities

A farmer keeps an inherited gold donkey in his stable which produces gold in wondrous way. After all costs are paid, the gold donkey produces $1,000 each year. Another fantastic attribute of the gold donkey is its immortality. The farmer knows that a gold donkey of this kind has got to be high in value. The price of the gold donkey would be infinitely high. So he tries to sell the gold donkey. The farmer invites many offers, but no offer was higher than $50,000. The farmer asks a potential buyer why he didn't offer more than $50,000 for the gold donkey. What is the reason?

If the buyer invest for example $60,000 on bank account with the current market interest rate of 2% per annum, he will get back $1,200. However the gold donkey produces only $1,000 per annum. Thus it is more favorable to put $60,000 into a bank account instead of buying a gold donkey.

If the buyer invests $40,000 in a bank account, then he receives $800 per annum with the current market interest rate of 2% per annum. The gold donkey produces, however, €1,000 per annum. Thus, it is more favorable to buy a gold donkey for €40,000 instead of investing the money in a bank account.

But what is the actual value of this fabulous gold donkey? We can use the following formula:

$$Present\ value\ of\ perpetuity = \frac{C}{r}$$

$$Present\ value\ of\ perpetuity = \frac{1{,}000}{0.02} = \$50{,}000$$

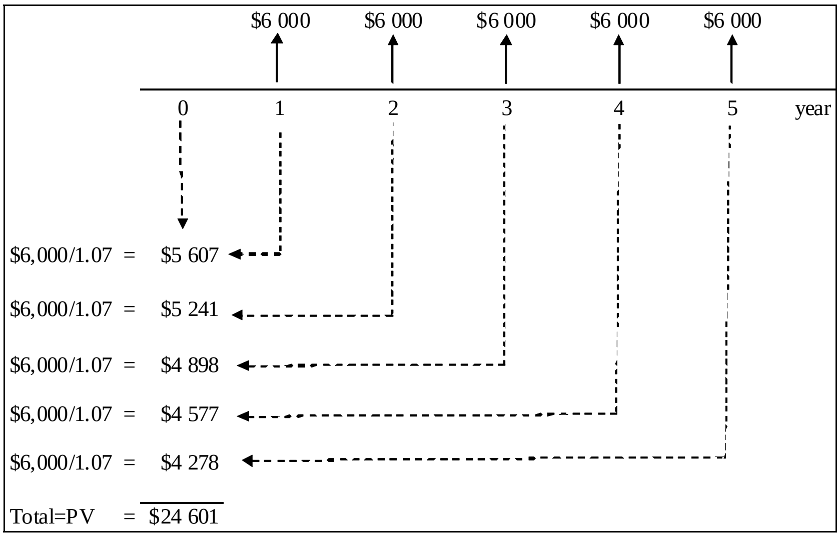
Annuities

An annuity produces a fixed sum each year for a specified number of years. Examples of annuities are an installment credit agreement or the equal-payment house mortgage. The general formula to value annuity that pays $1 a year for each of t years starting in year 1 is:

$$\text{Present value of annuity} = \frac{1}{r} - \frac{1}{r(1+r)^t}$$

Example: Present value annuities

Volvo offers a special offer for their customers. When you buy a new Volvo you pay $6,000 at the end of each of the next five years and no cash down. The interest rate is 7%. How much money does the car really cost? To solve the problem we could use our standard discounted cash flow formula (DCF).



Calculation showing the year-by-year present value of payments

But we have learned a formula for handling this kind of problem:

$$\text{Present value of annuity} = \frac{1}{r} - \frac{1}{r(1+r)^t}$$

$$PV = 6{,}000 \times \left[\frac{1}{0.07} - \frac{1}{0.07(1.07)^5} \right] = 6{,}000 \times 4.10 = \$24{,}601$$

You can find the annuity factors at the end of your workbook (Appendix A, Table 3).

Formulary of perpetuities and annuities

	Year: 1	2	3	4	5		Present Value
1. Three-year annuity	$1	$1	$1				$\frac{1}{r} - \frac{1}{r(1+r)^3}$
2. Perpetuity A	$1	$1	$1	$1	$1	...	$\frac{1}{r}$
3. Perpetuity B			$1	$1	$1	...	$\frac{1}{r(1+r)^3}$

Important formulas

Present Value Annuities Due

When we use the standard annuity formula, we assume that the first payment was made at the end of year one. But what will happen when the first payment is at the beginning of year one? An annuity due is worth (1+r) times the value of an ordinary payment.

$$\text{Present value of annuity due} = (1+r)\left(\frac{1}{r} - \frac{1}{r(1+r)^t}\right)$$

Annual Payments: Example

We receive a mortgage loan. The present value of the loan is $300,000 and the bank requires us to repay it in equal annual installments over the next 20 years.

$$PV = \text{mortgage payment} \times 20\ \text{year annuity factor} = \$300{,}000$$

$$\text{Mortgage payment} = \$300{,}000 / 20\ \text{year annuity factor}$$

Suppose that the interest rate is 14%...

$$20\ \text{year annuity factor} = \left[\frac{1}{0.14} - \frac{1}{0.14(1.14)^{20}}\right] = 6.623$$

$$\text{Mortgage payment} = \$300{,}000 / 6.623 = \$45{,}297$$

This example is an amortizing loan. This means that part of the regular payment rate is used to pay for reduction and the other part for interest.

Future value of an annuity: Example

When we want to calculate the future value of a level stream of payments, we have to calculate the present value and then multiply by $(1+r)^t$. The formula is…

$$Future\ value\ of\ annuity = \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \times (1+r)^t = \frac{(1+r)^t - 1}{r}\right]$$

In this example, we think about buying a new car in 5 years. We know that we can save $5,000 every year and we earn 8% interest on these savings. So how much money do we have after saving for 5 years?

As first we use the well-known formula for determining the present value of an annuity. The present value is…

$$PV = \$5{,}000 \times 5\, annuity\ factor$$
$$PV = \$5{,}000 \times 3.993 = \$19{,}965$$

Now we have to think about how much money we have if we invest $19,965 today.

$$Value\ of\ the\ end\ of\ year\ 5 == \$19{,}965 \times 1.08^5 = \$29{,}335$$

After 5 years we should be able to buy a nice car.

More Shortcuts: Growing Perpetuities and Annuities

Often we have to value cash flows that have constant growth. In this section, you will see some shortcuts to value perpetuities and annuities. In the overview, **g** is the growth rate. The following overview shows you the rule for growing perpetuities and annuities.

Year:	1	2	3	4	5	...	Present Value
1. Growing Three-year annuity	$1	$1x(1+g)	$1x(1+g)^2				$\frac{1}{r-g} - \frac{1}{(r-g)(1+r)^3}$
2. Growing Perpetuity A	$1	$1x(1+g)	$1x(1+g)^2	$1x(1+g)^3	$1x(1+g)^4	...	$\frac{1}{r-g}$
3. Growing Perpetuity B				$1x(1+g)^3	$1x(1+g)^4	...	$\frac{1}{(r-g)(1+r)^3}$

Formulary

Compendium with all relevant formulas for exercises.

Cash flow, $							
Year:	0	1	2...	...t-1	t	t+1...	Present Value
Perpetuity		1	1...	1	1	1...	$\frac{1}{r}$
t-period annuity		1	1...	1	1		$\frac{1}{r}-\frac{1}{r(1+r)^t}$
t-period annuity due	1	1	1...	1			$(1+r)(\frac{1}{r}-\frac{1}{r(1+r)^t})$
Growing perpetuity		1	1x(1+g)	$1x(1+r)^{t-2}$	$1x(1+g)^{t-1}$	$1x(1+g)^{t}$...	$\frac{1}{r-g}$
t-growing annuity		1	1x(1+g)	$1x(1+g)^{t-2}$	$1x(1+g)^{t-1}$		$\frac{1}{r-g}-\frac{1}{r-g}\times\frac{(1+g)^t}{(1+r)^t}$